Notes on Indian Mathematics.

A criticism of George Rusby Kaye's interpretation.

The following lines occur in VINCENT SMITH's *The Early History of India* (3rd edition, 1914, page 305; 4th edition, 1924, revised by S. M. EDWARDES, pp. 322 & 323) :

« Mr. G. R. KAYE, a competent authority, holds that 'the period when mathematics flourished in India commenced about A.D. 400 and ended about A.D. 650, after which deterioration set in.' »

In his *A Short History of Mathematics* Dr. FLORIAN CAJORI writes that he has drawn heavily upon Mr. G. R. KAYE's *Indian Mathematics* (Calcutta, 1915) to write the chapter on Indian mathematics (1).

Dr. DAVID EUGENE SMITH and Sir THOMAS HEATH — two well-known historians of mathematics — have based some of their conclusions regarding Indian mathematics on the writings of Mr. G. R. KAYE.

Besides the late Messrs VINCENT SMITH and S. M. EDWARDES Sir RICHARD TEMPLE describes Mr. G. R. KAYE as an authority on Indian astronomy (2).

The above facts show in what light Mr. KAYE's writings have been accepted by foreign scholars, both European and American. Accordingly many erroneous conclusions, like the one contained in the quotation with which this paper opens, promulgated by

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(1) Second edition, 1922, p. 84, foot-note.

This paper, *Indian Mathematics*, had been originally written for *Isis*, and the author had already read the proofs of it in April, 1914. Then publication was postponed by the war. The author became impatient and caused his paper to be reprinted and published independently (Calcutta 1915) with a few additions (articles 21 and 29, appendix 1, chronology, and index), but without any reference to *Isis*. The original paper appeared in *Isis* as soon as publication was resumed (vol. 2, 326-57, 1919). The following references to *Indian Mathematics* quote he pages of both the booklet and of *Isis* (Editor's Note).

(2) *Indian Antiquary*, Vol. 50 (February, 1921), p. 64.

Mr. KAYE regarding Indian mathematics and astronomy, have found a place in the works of foreign authors.

It may be stated at the outset that these foreign authors are not to blame for incorporating Mr. KAYE's conclusions in their works. Not being scholars of the Sanskrit language in which the works on Indian mathematics and astronomy were written, they had to depend on others for their knowledge of these subjects. Mr. G. R. KAYE's long residence in India as a high Government official, the association of his name with the English translations of ÂRYABHAṬA's *Gaṇitapâda* and ŚRÎDHARA's *Triśatikâ*, his copious writings on Indian mathematics and astronomy, and his condemnation (3) of previous orientalists who held views contrary to his own, together with the fact that his opinions and statement of facts long went without sufficient challenge (4) from the Indians, naturally induced foreign scholars to regard him as a competent authority on Indian mathematics and astronomy and to reject the conclusions of the previous orientalists in favour of those held by him. Before an opinion which is hitherto generally accepted is allowed to be replaced by a new one, the latter ought to be subjected to a severe test. But, unfortunately for the cause of Science, Mr. KAYE's conclusions have been allowed to supersede or modify the current ones without sufficient scrutiny. The cause of the history of Science, therefore, demands a careful and thorough examination of his methods of investigation and conclusions.

There is not the shadow of a doubt that Mr. KAYE is an able writer and has the gift of presenting even a weak case in an apparently convincing form. Yet, the present writer feels that, instead

(3) Mr. KAYE writes that STRACHEY, BURGESS, and TAYLOR « are most unreliable » (*JRAS*, 1910, p. 756). Again he writes (*JRAS*, 1911, p. 812) : « BOMBELLI stated that DIOPHANTUS often quotes from Indian authors. Such misrepresentations are so obviously wrong that they are readily detected; but COSSALI, Sir WILLIAM JONES, PLAYFAIR, TAYLOR, COLEBROOKE, ROSEN, LIBRI, MAX MULLER, and others are no less culpable and often their statements are all the more dangerous by being somewhat less startlingly false. » The present writer will feel amply gratified if this paper serves to rescue the names of these departed savants from the unjust aspersions of Mr. G. R. KAYE.

(4) Mr. NALIN BIHARI MITRA's challenge published in the *Modern Review* (Calcutta) for July, August, and November, 1915 and also for June, 1916, does not seem to have reached foreign scholars of mathematics. Mr. N. K. MAZUMDAR's challenge published in the *Bulletin* of the Calcutta Mathematical Society, Vol. III (1911-12), does not seem to have produced any effect as yet.

of advancing the cause of Science, he has, by his writings, done considerable harm to it and that the history of mathematics and astronomy, at least so far as it relates to India, would have to be re-written.

If, as is alleged (5) by Mr. KAYE, « the tendency of the early orientalists was towards antedating » and they « were not always perfect in their methods of investigation », his own tendency has been towards postdating and his methods of investigation are open to serious objections. Consider the following instances taken from his *Indian Mathematics* :

(I) « *The word-symbol notation.* — A notation that became extraordinarily popular in India and is still in use was introduced about the ninth century, possibly from the East (6) » (page 31, lines 4-6; *Isis*, 2, 345).

(II) « There is no sound evidence of the employment in India of a place-value system earlier than about the ninth century » (page 31, lines 28-30; *Isis*, 2, 346).

(III) « The proof by 'casting out nines' ... ... occurs in no Indian work before the 12th century » (page 34, lines 8 & 9; *Isis*, 2, 348).

(IV) The earliest example of an alphabetic system of notation based on the place-value idea is of the twelfth century. (Page 31, lines 1-3; *Isis*, 2, 345).

(V) ÂRYABHAṬA's value of π « was not used by any other Indian mathematician before the 12th century » (pages 12 & 13). Again, « the Indians record an extremely accurate value (i. e., of π) at a very early date but seldom or never actually use it (page 32, lines 21-23; *Isis*, 2, 344, 346.)

A reference to VARÂHAMIHIRA's (died 587 A.D.) *Pañcasiddhântikâ* (Text edited by THIBAUT and DVIVEDI) and BRAHMAGUPTA's (born 598 A.D.) works will conclusively prove that the first (7) of the above statements is entirely wrong.

(5) *Indian Mathematics* (Calcutta), 1915, p. 1, *Isis*, 2, 326.

(6) Mr. KAYE omits to mention the name of the country from which the notation is alleged to have been introduced into India.

(7) The fact that Mr. KAYE could make this statement in spite of his supposed intimate acquaintance with the works of VARÂHAMIHIRA and BRAHMAGUPTA proves that he never cared to consult the original Sanskrit texts and that his only source of information was the English translation thereof. To attract the attention of foreign scholars to Sanskrit expressions for numbers in terms of word-numerals THIBAUT and DVIVEDI have got them printed in Devanâgarî numerals over the

The second statement is also equally wrong. Competent authorities like BURNELL, BURGESS, and BÜHLER are of the opinion that the numerous examples of the word-numeral notation that occur in the works of VARÂHAMIHIRA, BRAHMAGUPTA and their successors presuppose the previous existence of the modern place-value decimal notation. I have elsewhere (8) shown that the elder ÂRYABHAṬA (born 476 A.D.) has recorded a brief enunciation of the modern place-value decimal notation in the second verse of the second chapter of the *Âryabhaṭîyam*. The following passage occurs in the *Vyâsa Bhâṣya* (*Bibhûtipâda*, 13th *sûtra*) of the *Yogasûtra* of PATAÑJALI :

« Yathaikâ rekhâ śatasthâne śataṃ daśasthâne daśa ekañcaikasthâne » (9) etc.

It may be translated thus :

« Just as the same stroke (or figure) represents a hundred, a ten, or a unit according as it is in the hundred's place, the ten's place, or the unit's place », etc.

Professor J. H. WOOD in this connection observes (10) that contrary to Mr. G. R. KAYE's opinion « the place system of decimals was known as early as the sixth century A.D. »

ŚAṄKARA's (c. 750) commentary on *Brahmasûtra* (i.e. *Vedântasûtra*) contains the following passage :

« Yathâ caikâpi satî rekhâ sthânânyatvena niveśyamânaikadaśaśatasahasrâdipratyayabhedamanubhabati » etc. (*Śârîrikabhâṣya*, 2.2.17).

It may be translated thus :

« Just as the same stroke (or figure) conveys different ideas such as a unit, a ten, a hundred, a thousand, &c., according to the place in which it is set down », &c.

The use of such passages for the purpose of elucidation of abstruse philosophical doctrines shows that the place-value decimal notation was extremely popular at the time when the above commentaries were written. No authority has placed ŚAṄ-

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number-expressions occuring on page 2 of the text of Pañca-siddhântikâ. Even these expressions do not seem to have been noticed by Mr. KAYE.

(8) *The American Mathematical Monthly* (October, 1927), pp. 409-15.

(9) My attention was kindly drawn to this passage and the next one by Dr. BIBHÛTIBHÛṢAN DATTA of the Calcutta University.

(10) Prof. J. H. WOOD's *English Translation of the Yoga System* of PATAÑJALI (Harvard Oriental Series). p. 216, foot-note.

KARA later than the latter half of the 8th century A.D. (*Encyclopædia of Religion and Ethics*, Vol. II. p. 185). Taking into consideration the absence of facilities of communication in those early times, it would not be extravagant to suppose that the decimal system of notation was at least two or three centuries old at the time of ŚAṄKARA. In no circumstances can the earliest date of the use of the modern place-value decimal system of notation in India be shifted to so late a period as the ninth century A.D.

The third statement also is untenable. The method of verification of the operations of multiplication, division, and extraction of square and cube roots by casting out nines (or more correctly, by repeated addition of digits) is given in the four concluding stanzas of the *Mahâsiddhânta* by the younger ÂRYABHAṬA who has been assigned to the middle of the tenth century by the late Mr. ŚAṄKARA BÂLAKRISHNA DIKSHIT (11) and also by Mr. R. SEWELL (12).

The younger ÂRYABHAṬA's *Mahâsiddhânta* will also disprove the fourth statement. This work practically opens with a statement of an alphabetic system of notation based on the place-value idea and contains numerous examples of this system.

The fifth statement has been commented upon by Dr. B. B. DATTA who has shown it to be wrong (13). The object of this statement is obviously to suggest the possibility of the value $\frac{62832}{20000}$ of π being given by the younger ÂRYABHAṬA. But the way in which the number 62832 has been expressed (namely, one hundred increased by four and then multiplied by eight, together with sixty-two thousand) shows that the author of the value cannot be the younger ÂRYABHAṬA or any other Indian mathematician who flourished after the middle of the sixth century A.D., since when, so far as our present knowledge goes, the method of expressing numbers in word-numeral notation based on the principle of place-value has come into use. The younger ÂRYABHAṬA uses his alphabetic (*kaṭapayâdi*) system of notation in his *Mahâsiddhânta* with the exception of the chapter on Arithmetic where he uses the popular word-numeral notation except

(11) *History of Indian Astronomy*, (Poona, 1896), p. 231.

(12) *The Siddhântas and the Indian Calendar*, (Calcutta, 1924), Preface, p. IX.

(13) See *Hindu Values of π*, *Journal of the Asiatic Society of Bengal*, New Series, 22, 39 & 40, 1926; see also pages 26 and 27.

in the cases of the four convenient numbers, 200, 250, 300, and 400. Hence, if the elder ÂRYABHAṬA had not been the author of the value of π under consideration, the complicated number which forms the numerator of the fractional value would have surely been expressed in the prevalent word-numeral notation. It might be stated here that the younger ÂRYABHAṬA gives the values $\sqrt{10}$ and $\frac{22}{7}$ of π and expresses the latter value in the current word-numeral notation.

Let us now examine Mr. G. R. KAYE's methods. The following examples will serve as illustrations :

(I) He almost correctly translates the elder ÂRYABHAṬA's principle of place-value as « each succeeding place is ten times the preceding » (14). Yet he states that « there is not in any part of ÂRYABHAṬA's work the remotest indication of a knowledge of a notation with 'place-values' » (15).

(II) MUHAMMED BEN MUSA gives the value of π exactly in the unsimplified form previously given by the elder ÂRYABHAṬA alone and BRAHMAGUPTA's rules for finding the circumference of a circle and the diameters of two intersecting circles when the common chord and the sagittae of the resulting arcs are given, without trying to show how these rules and the value of π can be obtained. He also gives the elder ÂRAYBHAṬA's rule for the area of a circle. Yet he observes that « it is absolutely certain that M. IBN MUSA did not copy his value of π from the Hindus » (16) and that « there is not the slightest resemblance between the previous Indian works and those of M. B. MUSA » (17).

(III) BRAHMAGUPTA attempted to solve the so-called Pellian equation and was only partially successful. It is BHÂSKARA who completed the solution. Yet Mr. KAYE writes (18): « BHÂSKARA gives some alternative methods for the solution of the Pellian equation, but in no essential does he improve on BRAHMAGUPTA ».

The above three cases are assertions which Mr. KAYE does not attempt to substantiate. He feels bound to make the third assertion in order to support his preconceived notion that deterio-

(14) *JASB*, March, 1908, p. 117.
(15) *JASB*, July, 1907, p. 494.
(16) *JASB*, March, 1908, p. 122.
17) *Indian Mathematics*, p. 42; *Isis*, 2, 354.
(18) *JASB*, 1910, p. 155.

ration in Indian mathematics took place before the time of BHÂSKARA.

(IV) In 1908 he published (19) a correct translation of ÂRYABHAṬA's rule, alleged to be the Greek rule *epanthema* in disguise. In 1915 he gave such a wrong translation of the same Indian rule as goes to show an apparent resemblance between ÂRYABHAṬA's rule and the Greek rule *epanthema* (20).

(V) To prove the alleged indebtedness of the Hindus to foreign sources he writes (21) : « BRAHMAGUPTA and BHÂSKARA distinctly indicate that they were compilers only, and frequent references are made by them to the 'text' and to 'ancient writers'. COLEBROOKE was misled into supposing that these ancient authorities were Hindus, but an examination of the references shews that the cases so referred to are just the cases that do not occur in earlier Hindu writings. »

The present writer believes that he has succeeded in showing elsewhere (22) that Mr. KAYE had never been at pains to undertake the examination of the references which he claims to have done.

(VI) To prove the alleged non-Indian origin of the modern arithmetical notation Mr. KAYE makes the following statement but does not give the reference :

« Sir RICHARD TEMPLE ... has shown us that the old ideas of notation still prevail, to a very great extent, among those in India who have not come in contact with foreign systems. This is practically the proof absolute that the new notation is not of Indian origin. » (23)

The statement attributed to Sir RICHARD TEMPLE appears to have been made by him with respect to Burma and not with respect to India (*Indian Antiquary*, 20, 53, 1891). On the other hand, he has distinctly stated that « a system corresponding to the European (*i.e.* the modern system of notation) has been in use among Hindus from a time long anterior to the era of British rule » (*ibid*). Sir RICHARD TEMPLE has also given the following reasons for the persistence of the old system :

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(19) *JASB*, March, 1908, p. 133.

(20) *Indian Mathematics*, p. 47 (not in *Isis*). For details see the *Journal of the Bihar and Orissa Research Society* for March, 1926, pp. 88-91.

(21) *JRAS*, 1910, p. 759.

(22) *Bulletin of the Calcutta Mathematical Society*, 18, 69-76, 1927.

(23) *JASB*, July, 1907, p. 494.

(a) It « is especially adapted to mental processes. »
(b) It « demands the least mental exertion compatible with calculating at all ».
(c) Under this system « it is not necessary to learn by rote to multiply beyond nine times nine ». (*Ibid*, p. 54).

But Mr. KAYE does not take these reasons into consideration in drawing an unwarranted conclusion.

(VII) Mr. KAYE writes : « ALBIRUNI tells us that BRAHMAGUPTA invented another system of notation generally designated by the term 'numerical words' » (*JASB*, July, 1907, p. 479). Let us compare this with the following statement of ALBIRUNI himself :

« BRAHMAGUPTA says : 'If you want to write *one*, express it by everything which is unique, as the *earth*, the *moon*; two by everything which is double, as, e. g. black and white;' (24) etc. (SACHAU's Translation of ALBIRUNI's, *India*, Vol. I, p. 177).

It will thus appear that ALBIRUNI never attributed the invention of the system of notation to BRAHMAGUPTA himself.

(VIII) To create a presumption against the Indian origin of the modern arithmetical notation in which the units increase from the right to the left Mr. KAYE states (25) that in India the smaller elements of a number used to be written first and refers his readers to the *Epigraphia Indica* as his authority. The following are some of the illustrations taken from that journal and altered to suit his purpose :

(1) *Karabâṇaviśvagaṇite* « i.e. reckoned by the hands (2), the arrow (50), and *visvas* (1300) or 1,352 (*Epigr. Ind.* v, 67) ». (*JASB*, July, 1907, p. 480).
(2) *Vedavasvagnicandra* « i.e. *vedas* (4), *vasus* (80), fires (300), the moon (1,000) or 1,384 (*Epigr. Ind.* i, 94) » (*Ibid*).
(3) *Yugakhendurûpa* « i.e. *yugas* (4), the sky (0), the moon (100), and the *rupa* (1,000) or 1,104 (*Epigr. Ind.* vi. 155) ». (*Ibid*).
(4) « *Tatvaloke* ($t = 6$, $v = 40$, $l = 300$, $k = 1000$, i.e. 1346) (*Epigr, Ind.* III, 40) » (*Ibid*, p. 479).

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(24) BRAHMAGUPTA's actual statement is as follows : « *Loke prasiddhasaṃjñâ rûpâdînâṃ śaśâṅkâdyâḥ* », or, 'The moon, etc., are the well-known names of one, etc.' (*Brahma-sphuṭa-siddhânta* edited by SUDHÂKAR DVIDEDI, Chap. XXIV, Verse 1).

(25) *JASB*, July, 1907, pp. 478 and 480.

(5) « Râghavâya ($r = 2, gh = 40, v = 400, y = 1000$ i.e. 1442) (*Epigr. Ind.* VI. 112) » (*Ibid*).

The writers in the *Epigraphia Indica* have actually used the words arrow, *viśva*, *vasu*, fire, moon and *rûpa*, and the Indian letters v, l, k, gh, and y in their accepted senses, namely, 5, 13, 8, 3, 1, 1, 4, 3, 1, 4, and 1 respectively and *not* as shown by Mr. KAYE. That Mr. KAYE has distorted the numerical significance of the word *moon* and of the letter v will appear even to a layman from the fact that the *moon* has been taken to signify both 1000 and 100 and v to signify both 40 and 400.

(IX) With the same object in view Mr. KAYE writes : « ÂRYABHAṬA's alphabetic notation ..... differed from the Brahmi notation in having the smaller elements on the left. » (*Indian Mathematics*, p. 30; *Isis*, 2, 344).

He cites two examples which support his statement but he omits the expressions *dhuṅvighva* ($= 140000 + 500 + 6000 + 4 + 60$) and *khricyubha* ($= 200 + 4000 + 60000 + 300000 + 24$) which do not support his view.

(X) To decry BRAHMAGUPTA Mr. KAYE writes : « A close examination of BRAHMAGUPTA's rules and examples establishes beyond all doubt that he was not their discoverer. He does not understand all the rules he gives. » (*JRAS*, 1910, p. 755).

It will appear from my paper *The source of the Indian solution of the so-called Pellian equation* (*Bulletin of the Calcutta Mathematical Society* for December, 1928) that Mr. KAYE either did not undertake a « close examination » of BRAHMAGUPTA's rules and examples or was quite unfit for the task.

(XI) In the mouth of a commentator Mr. KAYE puts the following remark regarding BRAHMAGUPTA :

« Having thus set forth the (solution of a) factum according to the doctrine of others, the author now delivers his own (incorrect) method with a censure on the other (correct method) » (*JASB*, March, 1908, p. 138).

Mr. KAYE has introduced the words 'incorrect' and 'correct method' only to cry down BRAHMAGUPTA. The commentator says nothing about the correctness or otherwise of the two methods (26).

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(26) *Algebra with Arithmetic and Mensuration from the Sanscrit of Brahmagupta and Bhascara* by COLEBROOKE, p. 362, foot-note 1.

In fact, both the methods are correct. For details the reader is again referred to my article *The source of the Indian solution of the so-called Pellian equation.*

(XII) In his article *The Source of Hindu Mathematics* published in the *Journal of the Royal Asiatic Society* for the year 1910 Mr. KAYE, like an impartial investigator, begins well by laying down the following three criteria (p. 750) :

(1) « The evolution of mathematical ideas cannot proceed *per saltum*, but must proceed in an orderly manner ».

(2) « While mathematical systems of independent growth will naturally have many points of similarity, yet differences are certain to occur; it is, indeed, impossible for two systems to grow up independently in exactly the same manner. »

(3) « Priority of statement of a proposition does not necessarily imply its discovery. »

Then he adds : « How far the Hindu system of mathematics satisfies such criteria remains to be seen. » (p. 751).

Only a detailed examination of Hindu mathematical works can show how far the above three tests are satisfied by Hindu mathematics. Whenever such an examination has been undertaken, the theory of foreign origin has been exploded. But Mr. KAYE always prefers to undertake a brief (27) examination of Hindu mathematics, points out some accidental superficial similarities which must exist between the Hindu system and the foreign systems, and, contrary to the second criterion formulated by himself, has no doubt as to the certainty of foreign origin of Hindu mathematics. Let us see how cleverly he avoids applying the tests which he himself has laid down. He qualifies his previous remark by saying : « Possibly in a matter like this any definite conclusion that may be formed will depend upon accumulative evidence. This is difficult to deal with rigorously, and we can formulate no criterion that will help us here; but we may point out that in this respect the opinions of experts are particularly valuable » (p. 751). But Mr. KAYE attaches no value to those opinions of experts which differ from his own preconceived ideas.

The above cases will suffice to show that his methods of

(27) *JASB*, June, 1908, p. 293; *JRAS*, 1910, p. 750; *Indian Mathematics*, p. 23; art. 19; *Isis*, 2, 341; *East & West* (Simla), July, 1918, pp. 673, 675, 679.

investigation cannot lead to sound conclusions and that he can scarcely be regarded as a competent authority on Indian mathematics. His writings have been responsible for many erroneous conclusions in the history of mathematics. It is time that the attention of scholars were directed to these errors with a view to their removal from history.

Let us next consider Mr. G. R. KAYE's statement as to the time when deterioration in Indian mathematics began to set in. He regards the senior ÂRYABHAṬA as a plagiarist or a borrower from foreign sources and BRAHMAGUPTA and his successors as compilers only. He also suspects that « there never was a school of Hindu mathematicians » (*JASB*, July, 1907, p. 496). Assuming for the sake of argument only that he is right in his estimate of Hindu mathematicians, we fail to see why the period during which mathematics is supposed by him to have flourished in India should include the senior ÂRYABHAṬA and BRAHMAGUPTA only and exclude MAHÂVÎRA, the younger ÂRYABHAṬA, ŚRÎDHARA, PADMANÂBHA and BHÂSKARA. So far as arithmetic and algebra are concerned, the topics dealt with by the senior ÂRYABHAṬA and BRAHMAGUPTA in their known works are less numerous than those given by MAHÂVÎRA, the younger ÂRYABHAṬA, and BHÂSKARA whose treatment is fuller and clearer. Therefore, even if we accept his estimate of Indian mathematicians, we are bound to admit that, if arithmetic and algebra flourished at the time of the senior ÂRYABHAṬA and BRAHMAGUPTA, they certainly did not cease to do so before the death of BHÂSKARA. It has been shown (28) that his estimate of Indian mathematicians is not correct and rests on a serious misreading of facts. MAHÂVÎRA, the junior ÂRYABHAṬA, and BHÂSKARA have made considerable improvements on the previously existing method of solution of indeterminate equations of the first degree. MAHÂVÎRA is the first Indian mathematician to record (29) a rule for finding the L. C. M. of given numbers. He utilises it in finding the sum and difference of simple fractions and also in giving a general solution of his own of simultaneous linear indeterminate equations. He has dealt with a number of new series and distinguished

(28) See the present writer's articles in the *Bulletin of the Calcutta Mathematical Society*, Vols. 18 and 19.

29) *Gaṇita-sâra-saṃgraha*, Chapter III, §§ 55 & 56.

between the positive and negative square roots of a positive number. The method of verification of the arithmetical operations by repeated addition of digits (i. e. by casting out nines) and the treatment of indeterminate analysis, as given by the junior ÂRYABHAṬA, cannot surely be regarded as instances of deterioration. The fact that BHÂSKARA at the end of his *Vîjagaṇita* acknowledges the algebraical works of BRAHMAGUPTA, ŚRÎDHARA, and PADMANÂBHA as his sources shows that ŚRÎDHARA and PADMANÂBHA were mathematicians of no mean merit. Unfortunately, these works of ŚRÎDHARA and PADMANÂBHA have not yet been recovered. BHÂSKARA's realisation of the true nature of the quotient of division by zero, his explanation of the method of false position, his admirable contribution to the theory of indeterminate analysis, both simple and quadratic, his anticipation of the modern idea regarding the convention of signs and of KEPLER's method of finding the surface and volume of a sphere by integration, and his important suggestion of the use of the letters of the Devanâgarî alphabet for unknown quantities in algebraic equations — which is the penultimate stage of what NESSELMANN calls Symbolic Algebra — cannot surely mark a period of deterioration.

Why, then, does Mr. KAYE fix 650 A.D. as the date when the period during which mathematics flourished in India ended in his opinion? In the Chronology given at the end of his *Indian Mathematics* he does not mention the name of any Indian mathematician or mathematical work between the years 150 B. C. and 476 A. D. Yet he includes the period A. D. 400 — 476 in the period of mathematical advancement in India. This fact makes his position all the more untenable.

In fixing the dates 400 A. D. and 650 A. D. he seems to have been guided, not by the actual state of mathematical knowledge in India but by a desire to establish his pet theory (subsequently recorded in his *Indian Mathematics* (30)) that India borrowed her entire mathematical knowledge from Greece and that the path of communication between the Indians and Greeks was by way of Persia. This theory requires that the period of mathematical advancement in India should fall within the Sássánian period (A. D. 229-652) (31) in Persia. To show that this require-

(30) Pages 44 & 45.
(31) *Ibid.* Chronology given at the end.

ment is fulfilled he elsewhere (32) quotes (or rather, misrepresents) two authorities, namely, CHASLES and COLEBROOKE. He makes the following quotation from CHASLES :

« L'ouvrage de BHÂSCARA n'est qu'une imitation très-imparfaite (33) de celui de BRAHMAGUPTA, qui y est commenté et dénaturé..... Les propositions les plus importantes de BRAHMAGUPTA... y sont omises, ou énoncées comme inexactes ..... Cette circonstance et les commentaires de différens scholiastes, nous paraissent prouver que, depuis BRAHMAGUPTA, les sciences, dans l'Inde, ont été en déclinant. »

This remark has been made by CHASLES, not with respect to the entire mathematical works of BHÂSKARA as Mr. KAYE wants his readers to believe but with regard to the geometrical portion of BHÂSKARA's works. The above extract has been taken from CHASLES' remarks on the Geometry of the Indians in his *Aperçu historique sur l'origine et le développement des Méthodes en Géométrie*, etc. The fact that BRAHMAGUPTA's important propositions on cyclic quadrilaterals find no place in subsequent Indian works has led CHASLES to conclude that the science of Geometry in India has been in decline after the time of BRAHMAGUPTA. Although BRAHMAGUPTA has made some very notable contributions to Geometry, this subject has never been the strong point of the Hindus. Except in a few cases Hindu Geometry was mainly experimental and intuitional (34). Hence, the Hindus never claim to have been great geometricians. But they do claim to have been the greatest calculators on the strength of their achievements in the fields of arithmetic, algebra and trigonometry. Although attempts have been made to deprive them of the position they have been rightly enjoying, these attempts have made their position stronger than ever.

Mr. KAYE cites the authority of COLEBROOKE as follows :

« COLEBROOKE says that ARYABHATA was superior to any Hindu who came after him and that deterioration rather than advancement

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(32) *JASB*, July, 1907, p. 496.

(33) This remark is not applicable to the arithmetical and algebraical works of BHÂSKARA.

(34) This goes to support the view that the Hindus were not aware, until at a much later date, of the demonstrative character of Greek geometry which, if known earlier, would have appealed readily to the speculative Hindu mind.

took place since the time of the more ancient author (p. 9) » (35).

Here also Mr. KAYE characteristically omits the subject of COLEBROOKE's remark so that the words 'deterioration' and 'advancement' may be made to apply to any branch of Hindu achievements. Let the reader compare the observation attributed to COLEBROOKE by Mr. KAYE with what the former actually states. COLEBROOKE writes : « ARYABHATTA appears to have had more correct notions of the true explanation of celestial phenomena than BRAHMAGUPTA himself; who in a few instances, correcting errors of his predecessor, but oftener deviating from that predecessor's juster views, has been followed by the herd of modern Hindu astronomers, in a system not improved, but deteriorated, since the time of the more ancient author. » (36)

Does COLEBROOKE here speak of deterioration in Hindu *mathematics* ?

It will appear from the above that Mr. KAYE's opinion quoted by the late Mr. VINCENT SMITH regarding the period of mathematical advancement in India is not correct and that the period did not come to an end until after the death of BHÂSKARA.

Enough has been written to show that Mr. KAYE's views regarding Indian mathematics are erroneous. Some of his views on Indian astronomy have been challenged by Messrs. P. C. SEN GUPTA and S. R. DAS. The present writer is not competent to express any opinion on this subject. M. NAU's statement that the Indian figures were known in Syria in A. D. 662 has elicited from Mr. KAYE the observation that « his (NAU's) authority makes such erroneous statements about 'Indian' astronomy that we have no faith in what he says about other 'Indian' matters. » (*Indian Mathematics*, p. 31). In Mr. KAYE's own words one might similarly say, « Mr. KAYE makes such erroneous statements about Indian mathematics that we have no faith in what he says about other Indian matters. »

Cuttack, India. SÂRADÂKÂNTA GÂṄGULI.

(35) *JASB*, July, 1907, p. 496.
(36) *Dissertation*, p. IX.